1. (11 points) An observer is 30 meters from a point on the ground directly below a vertically rising balloon, as in the illustration. When the balloon is 40 meters above the ground, it is rising at a rate of 3 meters per second. What is the rate of change of the direct distance between the observer and the balloon?

\[30^2 + y^2 = D^2\]
\[900 + y^2 = D^2\]
\[0 + 2y \frac{dy}{dt} = 2D \frac{dD}{dt}\]
\[\frac{dD}{dt} = \frac{2y \frac{dy}{dt}}{D}\]
\[240 = 100 \frac{dL}{dt}\]
\[\frac{dL}{dt} = \frac{240}{100} = \frac{2.4}{5} \text{ m/s}\]

2. (11 points) All of the edges of a cube have equal lengths. An edge is measured to be 3 ± 0.1 cm. Use differentials to approximate the maximum error in the volume.

\[V = \pi x^3\]
\[dV = 3\pi x^2 dx\]
\[= 3(3^2)(\pm 0.1)\]
\[= \pm 2.7 \text{ cm}^3\]
3. (9 points each) Find the value of the following integrals.

   a) \[ \int \left( 6x^2 + 8x - 4 + \frac{12}{x^4} \right) \, dx \]
   \[ = 6 \left( \frac{x^3}{3} \right) + 8 \left( \frac{x^2}{2} \right) - 4x + 12 \left( \frac{x^{-3}}{-3} \right) + C \]
   \[ = 2x^3 + 4x^2 - 4x - \frac{4}{x^3} + C \]

   b) \[ \int x^2(\sqrt{x} + 6) \, dx \]
   \[ = \int (\frac{x^{\frac{5}{2}}}{6} + 6x^2) \, dx \]
   \[ = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + 6 \left( \frac{x^3}{3} \right) + C \]
   \[ = \frac{2}{7} x^{\frac{7}{2}} + 2x^3 + C \]

   c) \[ \int (3x^4 - 1)^3 \, dx \]
   \[ \text{Let } u = 3x^4 - 1 \]
   \[ \frac{du}{dx} = 12x^3 \, dx \]
   \[ \frac{1}{12} \, du = x^3 \, dx \]
   \[ \int u^{\frac{5}{12}} \, du \]
   \[ = \frac{u^{\frac{6}{12}}}{\frac{6}{12}} + C \]
   \[ = \left( \frac{3x^4}{72} \right) - 1 \]
   \[ = \frac{3x^4 - 1}{72} + C \]

   d) \[ \int \frac{4x}{3x^2 - 5} \, dx \]
   \[ \text{Let } u = 3x^2 - 5 \]
   \[ \frac{du}{dx} = 6x \, dx \]
   \[ \frac{1}{6} \, du = x \, dx \]
   \[ \int \frac{\frac{1}{6} \, du}{u} \]
   \[ = \frac{1}{3} \ln |u| + C \]
   \[ = \frac{2}{3} \ln |3x^2 - 5| + C \]
4. (11 points) Suppose that the front of a glacier is initially 100 cm from a reference point and the glacier front is moving at a rate of $10e^{2t} + 6e^{-3t}$ cm per year. Find its distance from the reference point after $t$ years.

$$S(t) = \int (10e^{2t} + 6e^{-3t}) dt$$

$$= 10\frac{e^{2t}}{2} + 6\left(-\frac{1}{3}\right)e^{-3t} + C$$

$$= 5e^{2t} - 2e^{-3t} + C$$

$S(0) = 100$

$100 = 5e^{0} - 2e^{0} + C$

$100 = 3 + C$

$97 = C$

$$S(t) = 5e^{2t} - 2e^{-3t} + 97$$

5. (12 points) Approximate $\int_{1}^{2} \sqrt{x^2 + 1} \, dx$ using $n = 4$ rectangles and right endpoints.

$$\Delta x = \frac{2 - 1}{4} = \frac{1}{4} = .25$$

$$f'(x) = \sqrt{x^2 + 1}$$

$$\begin{array}{cccccc}
& 1 & 1.25 & 1.5 & 1.75 & 2 \\
\hline
1 & 1 & 1 & 1 & 1 & 1 \\
\end{array}$$

$$\int_{1}^{2} \sqrt{x^2 + 1} \, dx \approx f(1.25) \Delta x + f(1.5) \Delta x + f(1.75) \Delta x + f(2) \Delta x$$

$$= \sqrt{(1.25)^2 + 1} (1.25) + \sqrt{(1.5)^2 + 1} (1.5) + \sqrt{(1.75)^2 + 1} (1.75) + \sqrt{(2)^2 + 1} (2.25)$$


$$= 1.141$$
6. (7 points) Given the graph of \( f(x) \) in the accompanying graph, what is the value of \( \int_0^7 f(x) \, dx \)?

\[
\int_0^7 f(x) \, dx = A_1 + A_2 \\
= (3)(2) + \frac{1}{2}(4)(2) \\
= 6 + 4 \\
= 10
\]

7. (11 points) Suppose that the volume of sediment, in kiloliters, in a dam impoundment grows at the rate show in the accompanying graph. Use 4 rectangles and midpoints to approximate the integral of this function. Include the units. What does your answer represent?

\[ \Delta \kappa = \frac{b-a}{n} = \frac{7-0}{4} = \frac{7}{4} \]

\[
(19)(.5) + (19.8)(.5) + (20)(.5) + (19.6)(.5) \\
= 39.2 \text{ kL}
\]

TOTAL ACCUMULATION OF SEDIMENT
FROM \( t=0 \) TO \( t=2 \) YEARS